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## CV and LMXB Population Synthesis

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**Abstract.** I discuss population synthesis methods in the context of low-mass compact binaries. Two examples, both constraining the largely unknown strength of orbital angular momentum losses, illustrate the power and problems of such studies. For CVs, the “standard” disrupted magnetic braking model predicts that systems below the period gap are on average older than systems above the gap. The corresponding difference in the space velocity dispersion is testable by observations, independent of brightness-dependent selection effects. For LMXBs, the fraction of transients among short-period neutron star systems turns out to be an important diagnostic quantity constraining not only the angular momentum loss rate but also the kick velocity imparted to the neutron star at birth and the common envelope efficiency. Small kicks ( $\lesssim 100$  km/s), low efficiencies and weak magnetic braking are strongly favoured.

### 1. Introduction

A binary population synthesis study considers global properties of a certain binary class and tries to relate these to known (or assumed) global properties of the progenitor population, ideally ZAMS binaries, by following evolutionary channels forming binaries of this class. Evolutionary timescales are usually much too long to give rise to directly observable changes of binary parameters in a given system. The only way to test evolutionary theories against observations is to consider a large sample of binaries in a class at different evolutionary states and compare the observed properties of this sample with results from population synthesis calculations.

So, not surprisingly, it has become increasingly popular to supplement the presentation of observational results with “predictions” from binary synthesis studies, often in ignorance of the significant uncertainties involved. A population synthesis necessarily deals with a large number of parameters, rendering predicted absolute numbers in many cases meaningless. The strength of population synthesis studies lies in the differential comparison between suggested models, allowing one to test their sensitivity to different parameters. A further severe problem is the fact that selection effects may distort the observed picture of a binary class so much that it bears no resemblance to the true intrinsic population.

In the following I illustrate the power and problems of population synthesis models with two applications to low-mass compact binaries. These are either cataclysmic variables (CVs) with a white dwarf accretor, or low-mass X-ray binaries (LMXBs), with a neutron star or black hole accretor. The Roche-lobe-filling donor is a low-mass main-sequence or giant branch star. These binaries represent a long-lived, interacting species at the endpoint of a chain of complex progenitor phases, ideal for study with population synthesis methods.

## 2. The General Framework

The standard evolutionary channel leading to the formation of low-mass compact binaries involves a common envelope phase (CE) where the more massive star, the progenitor of the compact star, swells to giant dimensions, overflows its Roche lobe and engulfs the secondary. Orbital energy, extracted by dynamical friction, expels the envelope. If the initial orbit was wide enough the remnant core and the secondary survive the CE as a binary with a much smaller separation (see e.g. Iben & Livio 1993 for a review).

The ZAMS mass  $M_p$  of the giant determines the nature of the later compact binary. If  $M_p \lesssim 10M_\odot$  the remnant core will be a white dwarf, for  $M_p \gtrsim 10M_\odot$  a helium star that eventually explodes as a supernova (SN) to leave either a neutron star (NS) or, if the helium star is more massive, a black hole (BH). Survival of the binary in the case of a SN depends critically on the amount of ejected mass in the explosion and on kick velocity imparted to the collapsed object if the explosion is asymmetric. The smallest  $M_p$  which leads to BH rather than NS formation is uncertain (e.g. Portegies Zwart et al. 1997), but usually taken to be of order  $30 - 40M_\odot$ .

The detached post-CE evolution and the subsequent (stable) semi-detached evolution is driven either by orbital angular momentum losses (“j-driven”) such as gravitational radiation and magnetic braking (MB), or by the nuclear expansion of the secondary (“n-driven”). Essentially all systems with an initial secondary mass above  $\simeq 1.5M_\odot$  are n-driven. This is because the secondary’s nuclear time  $t_{\text{MS}}$  decreases with stellar mass  $M_2$  roughly as  $t_{\text{MS}} \propto M_2^{-3}$ , and angular momentum losses become inefficient for large  $M_2$ . An exception are those systems with large mass ratios  $M_2/M_1$  which are unstable to thermal-timescale mass transfer (e.g. supersoft sources), which therefore maintain mass transfer without driving.

## 3. Example I: The Age of CVs

Population synthesis calculations for CVs within the above framework have been performed by a number of authors (e.g. Politano 1996; de Kool 1992; Kolb 1993; Yungelson et al. 1997, and references therein; for a review see Kolb 1995, 1996). Most studies aim at an interpretation of the deficiency of systems with orbital period between 2 and 3 h, the “period gap” in the orbital period distribution. In the standard model for CV evolution (cf. King, this volume) the gap arises because systems are detached and do not appear as CVs as they evolve from 3 to 2 h. The systems are thought to detach in response to a sudden drop of the angular momentum loss rate (by a factor  $\gtrsim 5$ ). This could for example

occur if magnetic braking (MB), the dominant driving above the gap, ceases to be effective once the secondary becomes fully convective. Below the gap the much weaker gravitational wave emission is likely to be the only driving mechanism. The disparity between the evolutionary timescales above and below the gap is reflected in the mean mass transfer rate, hence accretion rate and mean luminosity: systems above the gap are brighter and evolve faster. Full population models show that the vast majority of the intrinsic CV population has evolved past the period gap (or already formed below the gap). Only  $\simeq 1\%$  of all CVs actually populate the period regime above the gap at the present Galactic epoch. However, in the observed sample this population imbalance is hidden because the greater brightness of the small fraction of systems above the gap strongly increases the detection probability. Modelling this observational selection is difficult, but the main effect is easily illustrated by a visual magnitude-limited sample which to first order mimics the actual observed sample. The resulting detection probability is almost independent of the transfer rate, and the sample has almost equal numbers of systems above and below the gap (e.g. Kolb 1996). We conclude that the observed ratio of the number of CVs above to the number below the gap does not test the difference in evolutionary timescales. The ratio is primarily determined by how many CVs already *form* with short or long orbital periods. The timescale difference is not testable by determining the absolute magnitudes of individual CVs either because the instantaneous mass transfer rate can deviate substantially from the secular mean (cf. King, this volume).

Recently, a viable test which is largely independent of brightness-dependent selection effects was proposed by Kolb & Stehle (1996), stimulated by the compilation of CV  $\gamma$ -velocities by van Paradijs et al. (1996).

It is well known and well understood from basic principles of stellar structure and evolution that the long-term evolution of CVs proceeds in a uniform way, such that for example the mean mass transfer rate at a given period is the same for all systems (with some scatter due to different white dwarf masses), whatever the initial period (Stehle et al. 1996). This uniformity means that the luminosity, and therefore the detectability, of a system is independent of its age. On the other hand, the evolutionary timescale gives a clear signature of the age: systems above the gap evolve much faster and must on average be much younger than systems below the gap. The calculation of the detailed age distribution is a typical population synthesis application (for details see Kolb & Stehle 1996). Most systems above the gap are found to be younger than 1.5 Gyr, whereas systems below the gap have an average age of 3–4 Gyr. Note that “age” means the time elapsed since formation of the ZAMS progenitor binary. Although CVs born at long periods typically evolve into the gap in less than  $10^8$  yr after turn-on of mass transfer as a CV, there is no contradiction in having a CV age of several Gyr above the gap; this simply means that the system spent most of its time “dormant” as a detached pre-CV.

In principle, the age distribution can be determined observationally via (1) the cooling age of the white dwarf, or (2) the dispersion of space ( $\gamma$ -) velocities. It is difficult to deduce the mean cooling age from the surface temperature of a WD, which is constantly heated by accretion, dwarf nova and nova eruptions (cf. Sion, this volume). Method (2) makes use of the relation between the age of a stellar ensemble and its space velocity dispersion, found empirically for nearby

disc stars (e.g. Wielen 1977, Wielen et al. 1992, Freeman 1993). Although the precise shape of this relation is disputed, the controversy centers on very old stars, a regime unimportant for our purposes. Adopting the assumption that CVs and CV progenitors obey the same age–velocity dispersion relation as single disc stars leaves option (2) as the more promising one.

Convolving the age distribution and the age–velocity dispersion relation yields the dispersion  $\sigma_\gamma$  of  $\gamma$ -velocities as a function of orbital period. The standard model predicts a significant difference between  $\sigma_\gamma$  for systems above the period gap (typically  $\simeq 15$  km/s) and below the period gap ( $\simeq 30$  km/s). However, the observed sample compiled by van Paradijs et al. (1996) does not confirm such a difference.

There are two possible resolutions of this disagreement. First, the intrinsic scatter of the observed sample might be larger than estimated. More accurate radial velocity data should clarify this. Preliminary observations dedicated to high-precision measurements of  $\gamma$ -velocities are already under way (Marsh et al. 1997). Second, the evolutionary theory, or some part of it, could be wrong. An obvious way to reduce the age difference between systems above and below the gap — while keeping the model for the period gap intact — is to assume that magnetic braking is much less effective at long orbital periods, while it has the “standard” strength for periods close to 3hr, i.e. is sufficient to drive the mass transfer rate  $\simeq 10^{-9} M_\odot/\text{yr}$  needed to cause a detached phase of the right width in period. As an example, in population models with the extreme assumption that spin-orbit coupling is too weak to enforce corotation in detached systems, so that MB does not operate in the pre-CV phase,  $\sigma_\gamma$  increases overall and the difference above and below the gap decreases to  $< 10$  km/s.

#### 4. Example II: The transient fraction in short-period neutron star LMXBs

The formation of neutron star LMXBs (NS-LMXBs) within the standard framework has been the focus of a number of population synthesis studies (Iben, Tutukov & Yungelson 1996; Terman, Taam & Savage 1996; Kalogera & Webbink 1996, 1997). However only recently has it become clear that the fraction of transients among short-period NS-LMXBs represents an important constraint for these models. The interpretation of soft X-ray transient outbursts as disc instability phenomena in analogy to dwarf nova outbursts of CVs places an upper limit on the mass transfer rate  $M$  in transients. If  $M$  is too large the disc is hot, hydrogen fully ionized even in the outer disc, and no instability occurs. For LMXBs this critical rate is significantly smaller than for CVs because in the former the strong irradiation from the central source heats the disc and keeps it stable even for a relatively small transfer (accretion) rate (van Paradijs 1996; King, Kolb & Burderi 1996; see also King, this volume). The effect is particularly marked for neutron star systems, and less severe for black hole systems (King, Kolb & Szuszkiewicz 1997).

The secular mean mass transfer rate in most n-driven LMXBs and all black hole LMXBs is low enough for disc instabilities to occur, while all j-driven NS-LMXBs with ZAMS donors would be persistently bright if magnetic braking (MB) had the same functional form as for CVs, with a strength tightly

constrained at 3 h orbital period from the width of the CV period gap. The corresponding transfer rate is so much larger than the critical rate for transient behaviour in NS-LMXBs that any reasonable “sub-standard” strength magnetic braking at longer periods would barely allow transients even at  $P \gtrsim 10$  h.

However, from observations it is clear that the fraction  $r$  of transients among short-period NS-LMXBs is non-negligible, although it is difficult to estimate the true *intrinsic* value of  $r$ . Three out of 15 NS (or suspected NS) systems with periods in the range  $3 < P/\text{h} < 18$  are classified as transient (Ritter & Kolb 1997). Most persistent sources in the Galaxy in this period range should have been detected by now, and most of the  $\simeq 30$  persistent sources with undetermined orbital period are probably short-period systems. Hence a fairly pessimistic lower limit for  $r$  is 5%. It is very likely that there are many more so far undetected short-period transients, perhaps more than 100, and that some of the  $\simeq 30$  known transients with undetermined periods are also short-period NS systems. The corresponding rather large value of  $r$  implies that the mass transfer rate in a large fraction of  $j$ -driven NS-LMXBs should be much smaller than the secular mean transfer rate in CVs. As fluctuations of the instantaneous mass transfer rate around the secular mean are probably unimportant in  $j$ -driven LMXBs (cf. King, this volume), the large transient fraction demands a small secular mean  $\dot{M}$  in these systems. The evolutionary state of the donor offers a promising explanation for this.

It is well-known that in  $j$ -driven evolution with a Skumanich-type magnetic braking formalism ( $\dot{J} \propto \Theta R^\rho \omega^3$ , where  $\Theta$  and  $R$  is the secondary’s moment of inertia and radius,  $\omega$  the binary angular velocity, and typically  $\rho \simeq 2 - 4$ ) the mass transfer rate is smaller if the donor is somewhat nuclear-evolved (e.g. Pylyser & Savonije 1988; Singer et al. 1993). “Somewhat” means that the secondary is still in the core hydrogen burning phase, but close to its end. To quantify this we define  $f = t_{\text{turn-on}}/t_{\text{MS}}$ , the age  $t_{\text{turn-on}}$  of the binary at turn-on of mass transfer in units of the secondary’s main-sequence lifetime  $t_{\text{MS}}$ . Then  $j$ -driven systems correspond to  $f < 1$ , and  $\dot{M}$  is smaller the closer  $f$  is to unity. The reason is twofold: first, an evolved secondary is “undermassive”, i.e. has less mass than a ZAMS star filling its Roche lobe at the same period would have. This in turn implies a smaller angular momentum loss rate as  $\dot{J} \propto M_2^{\rho/3}$  with  $R \propto M_2^{1/3}$  from Roche geometry. Second, evolved stars are more centrally condensed, hence the moment of inertia is smaller.

For simplicity we identify the fraction  $r$  of transients with the fraction  $r(f > f_0)$  of those systems in the population which have a secondary that is more evolved than  $f_0$ . Until detailed calculations of the secular evolution for LMXBs for a wider range of initial secondary masses, MB strengths and values of  $f$  are performed the critical value  $f_0$  allowing transient behaviour must be treated as a free parameter. We expect  $f_0 \gtrsim 0.8 - 0.9$  (e.g. Pylyser & Savonije 1988).

King & Kolb (1997) investigated the formation of LMXBs under the assumption of spherically symmetrical SN explosions and magnetic braking following the prescription of Verbunt & Zwaan (1981). They showed that  $j$ -driven NS-LMXBs can form only under very special formation conditions (cf. King, this volume) which require that the secondaries are fairly massive ( $\gtrsim 1.2M_\odot$ ) at mass transfer turn-on, and that indeed a large fraction of systems have  $f > f_0$ .

However, this fraction is smaller if the SN is asymmetric and the neutron star receives a kick velocity. Two effects of kicks matter most. First, binaries which would have been unbound by a symmetric SN, i.e. systems with relatively small secondary masses  $M_2$ , can survive the explosion if the kick is suitably directed. Hence the mean secondary mass in the post-SN binaries decreases with increasing kick speed. Second, in the presence of kicks the post-SN orbital separation after circularisation is on average smaller, so that the systems spend less time in the detached state. Both effects decrease the mean value of  $f$  in the population.

Evidence for the presence of significant kick velocities at the formation of neutron stars is substantial (e.g. Kaspi et al. 1996, Hansen & Phinney 1997, Lorimer et al. 1997, Fryer & Kalogera 1997), and the question arises if it is possible to form  $j$ -driven NS-LMXBs with moderate kick velocities but still maintain a large fraction  $r(f > f_0)$ . A natural way to “compensate” the reduction of  $r$  from kicks is to reduce the MB strength at longer periods. This keeps the systems longer in the detached post-SN state and allows the secondary to evolve.

Again, testing this is a typical application for population synthesis calculations. Below I present some results of a systematic study which makes use of the models by Kalogera & Webbink (1996, 1997). Details will be presented elsewhere (Kalogera, Kolb & King 1997).

Figure 1 shows the transient fraction  $r(f > f_0)$  as a function of  $f_0$  for various parameter combinations. The upper panels assume inefficient CE ejection (efficiency parameter  $\alpha_{CE} = 0.3$ , i.e. only 30% of the released orbital energy is used to unbind the envelope), and the lower panel efficient ejection ( $\alpha_{CE} = 1.0$ ). The kick velocity distribution is assumed to be Maxwellian, with a mean value  $\langle v_{kick}^2 \rangle^{1/2}$  of 20 km/s (left), 100 km/s (middle) and 300 km/s (right). The linestyle indicates the magnetic braking law. Solid lines correspond to “strong” braking (as in King & Kolb 1997), dashed lines to “weak” braking (as in Kalogera & Webbink 1997). Both MB descriptions are Skumanich-type laws, calibrated to the CV period gap, with  $\rho = 4$  and 2 in the strong and weak case. In addition, the weak case involves a  $M_2$ -dependent reduction factor which drops exponentially for  $M_2 > 1.0M_\odot$ .

The strong MB case with small kick and efficient CE ejection shown in the lower left panel confirms the result of King & Kolb (1997) for spherical symmetrical SNe: a large fraction of systems are close to  $f_0 = 1$ . In the corresponding model with weak MB,  $j$ -driven systems do not form in significant numbers, and the curve  $r(f_0)$  is not shown in this panel. This is due to constraints on the primary’s progenitor evolution. Roche lobe overflow must occur after core He burning but prior to the SN explosion. As the radius growth of massive stars in this phase is rather limited (Schaller et al. 1992) the allowed range of initial orbital separations is very narrow. Therefore the range of post-CE separations is narrow as well. If the CE ejection is efficient ( $\alpha_{CE}$  large) the orbits in this narrow range are generally too wide for the weak angular momentum losses to establish contact within a Hubble time (cf. Kalogera & Webbink 1997).

As expected, with increasing mean kick velocity the fraction of unevolved systems (small  $f$ ) in the population increases significantly, while the transient fraction ( $f \gtrsim 0.8$ ) decreases sharply. Because of the restricted range of post-

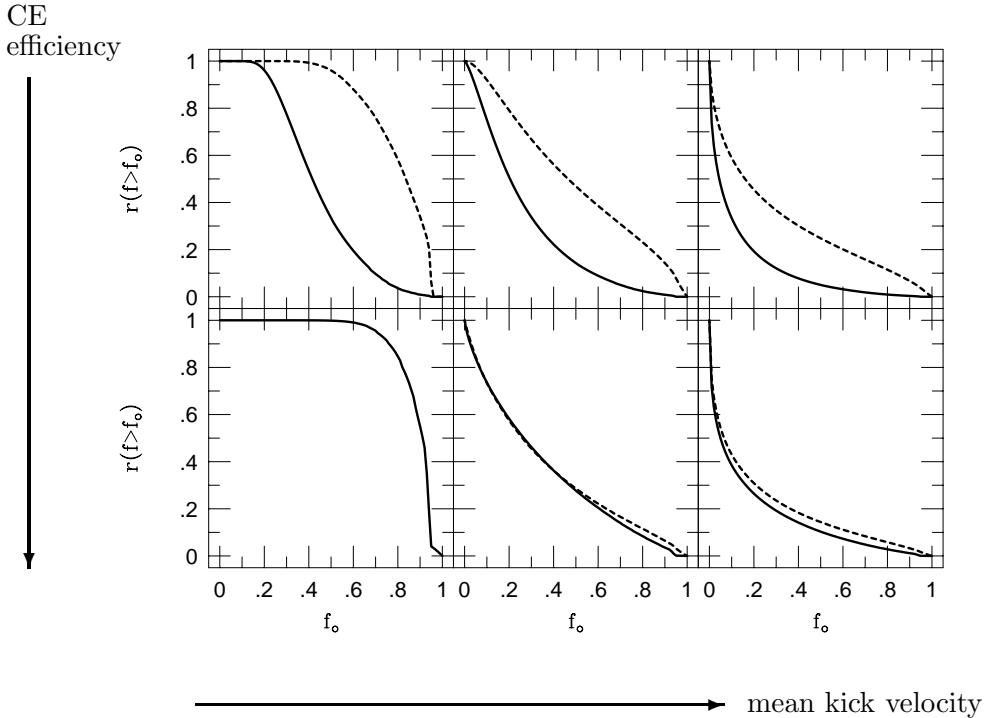


Figure 1. The transient fraction  $r(f > f_0)$  in short-period neutron star LMXBs as a function of the critical evolutionary state  $f_0$  for transient behaviour, for different model parameters. Full line: strong magnetic braking; dashed line: weak magnetic braking. Upper panels:  $\alpha_{CE} = 0.3$ , lower panels:  $\alpha_{CE} = 1.0$ . Left panels: mean kick velocity 20 km/s; middle panels: 100 km/s; right panels 300 km/s. See text for further details.

CE orbits the effect of changing  $\alpha_{CE}$  is more complex. The curves  $r(f_0)$  are insensitive to the MB strength for efficient CE ejection, while for inefficient CE ejection the transient fraction is generally larger in the case of weak MB. Comparing curves for the same MB law and  $\langle v_{\text{kick}}^2 \rangle^{1/2}$  but different  $\alpha_{CE}$  shows that in the weak MB case the transient fraction is larger for  $\alpha_{CE} = 0.3$  whereas in the strong MB case it is larger for  $\alpha_{CE} = 1$ .

With our estimates  $f_0 \simeq 0.8$  and  $r \gtrsim 0.3$ , and assuming that the kicks are not generally negligible, the models clearly favour small kick velocities, inefficient CE ejection and weak MB. In particular, it is hard to reconcile the necessity for a significant number of short-period transient NS-LMXBs with a mean kick velocity in excess of 100 km/s.

An important additional constraint on these parameters comes from the ratio of long-period (n-driven) to short-period (j-driven) NS-LMXBs. If the kicks are too small and MB too weak, too many n-driven systems would form, contrary to what is observed.

Taken together, tight constraints can be placed on all three parameters.

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